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Bianchi type-V string dust cosmological models with bulk viscous magnetic field

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Abstract— We investigate Bianchi Type-V magnetized bulk viscous sting cosmological model in general theory of relativity. To get determinate solutions, coefficient of bulk viscosity (ξ) is inversely proportional to the expansion (θ) is considered. The behavior of the model in presence and absence of magnetic field and bulk viscosity are discussed. The physical and geometrical aspects of the model are also investigated.

Keywords: Bianchi Type-V, Magnetic field, string cosmology, bulk viscosity

1 INTRODUCTION

It is a challenging problem to determine the exact physical situation at the very early stages of the formation of our universe. Among the various topological defect that occurred during the phase transition and before the creation of particles in the early universe, strings have interesting cosmological consequences and have been studied in more details (Vilenkin [1]). It is believed that cosmic strings give rise to density perturbations, which leads to the formation of galaxies (Zel'dovich[2]). Moreover, the magnetic field has important role at the cosmological scale and is present in galactic and intergalactic spaces. The importance of the magnetic field for various astrophysical phenomena has been studied by Banerjee et al.[3] , Chakraborty[4], Tikekar and Patel [5], Patel and Maharaj [6] and Singh and Singh[7] . Further Melvin [8] has pointed out that during the evolution of the universe, the matter was highly ionized state and is smoothly coupled with the field, subsequently forming neutral matter as a result of expansion of the universe.

Bianchi Type-V space times are interesting to study because of their richer structure both physically and geometrically than standard Friedman Robertson- Walker (FRW) models. These models represent the open FRW cosmological model with $k=-1$,where k is the curvature of three dimensional spaces at any time. Nayak and Sahoo[9] have investigated Bianchi Type-V model for a matter distribution admitting anisotropic pressure and heat flow. Ram [10] has obtained Bianchi Type –V cosmological model for perfect fluid distribution. He has given a new method to generate exact solution of Einstein field equation in Bianchi Type –V space- time.

Viscosity is important for number of reasons. Heller and Klimek [11] have investigated viscous fluid cosmological model without initial singularity. They have shown that the introduction of bulk viscosity effectively remove the initial singularity .Roy and Singh [12] have investigated LRS Bianchi Type-V cosmological model with viscosity, Santos et al.[13] investigated isotropic homogeneous cosmological model with bulk viscosity assuming viscous coefficient as power function of mass density. Banerjee and Sanyal [14] have investigated Bianchi Type-V cosmological models with viscosity and heat flow. Coley [15] investigated Bianchi Type V imperfect fluid cosmological models for equations of state $p=(\gamma-1)\rho$ where ρ is the energy density, p the pressure and $0\le\gamma\le2$. Bali and Yadav[16] have investigated an LRS Bianchi Type –V viscous fluid cosmological model assuming the condition $\sigma\propto\theta$, where σ is the shear and θ the expansion in the model. Bali and Singh [17] have investigated Bianchi Type –V viscous fluid string dust cosmological model assuming the condition that the bulk coefficient (ξ) is inversely proportional to the expansion (θ) in the model and shown the existence of the model.

In this paper, we have investigated Bianchi Type-V magnetized bulk viscous string dust cosmological model. It has been shown that the string dust cosmological model in the presence of bulk viscosity is not possible. The physical behavior of the model in the presence and absence of magnetic field and bulk viscosity is also discussed.

Field equation and their solutions:

We consider the Bianchi type-V metric in the form

$$ds^2 = -dt^2 + a_1{}^2 dx^2 + a_2{}^2 e^{-2x} dy^2 + a_3{}^2 e^{-2x} dz^2$$
(1)

where a_1, a_2 and a_3 are function of t only.

The energy momentum tensor for the bulk viscous string with, magnetic field is taken as

$$T_i^{\ j} = \rho u_i u^j - \lambda x_i x^j - \xi\theta\left(u_i u^j + g_i^{\ j}\right) + E_i^{\ j} \qquad (2)$$

with

$$u_i u^j = -x_i x^j = -1 \qquad and \qquad u_i x^j = 0 \qquad (3)$$

where ρ being the rest energy density of the system of strings, λ the tension density of the strings which may be positive, negative and zero as well, u^i the four velocity vector and x^i the direction of strings $\theta = u^i{}_{;i}$ is the scalar of expansion and ξ the coefficient of bulk viscosity. Here $E_i^{\ j}$ is the electromagnetic field given by

$$E_i^{\ j} = \frac{1}{4}\left(F_{j\alpha} F^{i\alpha} g^{\alpha\beta} - \frac{1}{4} g_i^{\ j} F^{\alpha\beta} F_{\alpha\beta}\right) \qquad (4)$$

where F_{ij} is the electromagnetic field tensor which satisfies the Maxwell equation

$$F_{[ij;\alpha]} = 0 \qquad \left(F^{ij}\sqrt{-g}\right)_{;\, j} = 0 \qquad (5)$$

In commoving co-ordinates, the incident magnetic field is taken along x-axis, with the help of Maxwell equation (5), the only non-vanishing component of F_{ij} is the equation

$$F_{23} = cons\tan t = A \qquad (6)$$

The Einstein field equations

$R_i^{\ j} - \frac{1}{2} R g_i^{\ j} = -T_i^{\ j}$ (using the units in which C=G=1) (7)

For metric (1) leads to

$$\frac{-3}{a_1{}^2} + \frac{\dot a_1 \dot a_2}{a_1 a_2} + \frac{\dot a_1 \dot a_3}{a_1 a_3} + \frac{\dot a_2 \dot a_3}{a_2 a_3} = \rho + \frac{A^2}{8\pi a_2{}^2 a_3{}^2} \qquad (8)$$

$$-\frac{1}{a_1{}^2} + \frac{\ddot a_2}{a_2} + \frac{\ddot a_3}{a_3} + \frac{\dot a_2 \dot a_3}{a_2 a_3} = \lambda + \frac{A^2}{8\pi a_2{}^2 a_3{}^2} + \xi\theta \qquad (9)$$

$$-\frac{1}{a_1{}^2} + \frac{\ddot a_1}{a_1} + \frac{\ddot a_3}{a_3} + \frac{\dot a_1 \dot a_3}{a_1 a_3} = -\frac{A^2}{8\pi a_2{}^2 a_3{}^2} + \xi\theta \qquad (10)$$

$$-\frac{1}{a_1{}^2} + \frac{\ddot a_1}{a_1} + \frac{\ddot a_2}{a_2} + \frac{\dot a_1 \dot a_2}{a_1 a_2} = -\frac{A^2}{8\pi a_2{}^2 a_3{}^2} + \xi\theta \qquad (11)$$

$$\frac{2\dot a_1}{a_1} - \frac{\dot a_2}{a_1} - \frac{\dot a_3}{a_3} = 0 \qquad (12)$$

where

$$\theta = \frac{3\dot a_1}{a_1} \qquad (13)$$

From equation (12), we obtain

$$a_1{}^2 = l a_2 a_3 \qquad (14)$$

where l is the constant of integration.

Equations (10) and (11) lead to

$$\frac{\ddot a_3}{a_3} - \frac{\ddot a_2}{a_2} + \frac{\dot a_1}{a_1}\left(\frac{\dot a_3}{a_3} - \frac{\dot a_2}{a2}\right) = 0 \qquad (15)$$

Using equation (12) in equation (15), we get

$$\frac{(\dot a_2 a_3 - a_2 \dot a_3)^{\bullet}}{\dot a_2 a_3 - a_2 \dot a_3} = -\frac{1}{2}\frac{(a_2 a_3)^{\bullet}}{a_2 a_3} \qquad (16)$$

This after integration leads to

$$a_3{}^2\left(\frac{a_2}{a_3}\right)^{\bullet} = \frac{L}{\sqrt{a_2 a_3}} \qquad (17)$$

where L is constant of integration .

Using equation (12) in equation (11), we get

$$2\frac{\ddot a_2}{a_2} + 6\frac{\ddot a_3}{a_3} - \frac{\dot a_2{}^2}{a_2{}^2} + \frac{a_3{}^2}{a_3{}^2} + 4\frac{\dot a_2 \dot a_3}{a_2 a_3} - \frac{4}{l a_2 a_3} = 4\xi\theta - \frac{4k}{a_2{}^2 a_3{}^2} \qquad (18)$$

Where

$$k = \frac{A^2}{8\pi} \qquad (19)$$

Let

$a_2 a_3 = \mu$ and $\frac{a_2}{a_3} = \upsilon$ (20)

Equations (17) and (18) with the help of (20) gives

$$\frac{\dot\upsilon}{\upsilon} = L\mu^{-3/2} \qquad (21)$$

and

$$4\frac{\ddot\mu}{\mu} - 2\frac{\ddot\upsilon}{\upsilon} - \frac{\dot\mu^2}{\mu^2} + 3\frac{\dot\upsilon^2}{\upsilon^2} - 3\frac{\dot\mu\dot\upsilon}{\mu\upsilon} - \frac{4}{l\mu} = 4\xi\theta - \frac{4k}{\mu^2} \qquad (22)$$

Using equation (21) in (22), we get

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$$\ddot{\mu}-\frac{\dot{\mu}^2}{4\mu}+\frac{L^2}{4\mu^2}-\frac{1}{l}=\xi\theta\mu-\frac{k}{\mu} \qquad (23)$$

To get a determinate model we assume that coefficient of bulk viscosity ξ is inversely proportional to expansion (θ) , thus we have

$$\xi\theta = const = M \qquad (24)$$

Thus equation (23) leads to

$$\ddot{\mu}-\frac{\dot{\mu}^2}{4\mu}=\frac{1}{l}+M\mu-\frac{L^2}{4\mu^2}-\frac{k}{\mu} \qquad (25)$$

Let $\dot{\mu}=f(\mu)$, then equation (25) lead to

$$\frac{d}{d\mu}f^2-\frac{f^2}{2\mu}=\frac{2}{l}+2M\mu-\frac{L^2}{2\mu^2}-2\frac{k}{\mu} \qquad (26)$$

From equation (26),we have

$$f^2=\frac{4\mu}{l}+\frac{4M}{3}\mu^2+\frac{L^2}{3\mu}+R\sqrt{\mu}+4k \qquad (27)$$

where R is constant of integration.
Equations (21) and (27) lead to

$$\log\upsilon=\int\frac{L\mu^{-3/2}d\mu}{\sqrt{\frac{4\mu}{l}+\frac{4M\mu^2}{3}+\frac{L^2}{3\mu}+R\sqrt{\mu}+4k}} \qquad (28)$$

Hence the metric (1) reduces to the form

$$ds^2=-\frac{d\mu^2}{\sqrt{\frac{4\mu}{l}+\frac{4M\mu^2}{3}+\frac{L^2}{3\mu}+R\sqrt{\mu}+4k}}+l\mu dx^2+\mu\upsilon e^{-2x}dy^2+\frac{\mu}{\upsilon}e^{-2x}dz^2 \qquad (29)$$

Using suitable transformation

$$\mu=T, x=X, y=Y, z=Z \qquad (30)$$

Using equation (30) in (29), we get

$$ds^2=-\frac{dT^2}{\sqrt{\frac{4T}{l}+\frac{4MT^2}{3}+\frac{L^2}{3T}+R\sqrt{T}+4k}}+lTdX^2+T\upsilon e^{-2X}dY^2+\frac{T}{\upsilon}e^{-2X}dZ^2 \qquad (31)$$

Here υ can be determined by (28).

Hence the energy density ρ and string tension λ for the model (31) are given by

$$\rho=\frac{2k}{T^2}+\frac{3R}{4T^{3/2}}+M \qquad (32)$$

$$\lambda=-\frac{2k}{T^2} \qquad (33)$$

The scalar of expansion (θ), shear (σ) and the spatial volume V^3 for the model (32) are given by

$$\theta=\frac{3}{2T^{3/2}}\left(\frac{4T^2}{l}+\frac{4MT^3}{3}+\frac{L^2}{3}+RT^{3/2}+4kT\right)^{\frac{1}{2}} \qquad (34)$$

$$\sigma^2=\frac{3}{4}L^2T^{-3} \qquad (35)$$

$$V^3=lT^2e^{-2x} \qquad (36)$$

In the presence of bulk viscosity and in the absence of magnetic field i.e. $k=0$, the energy density (ρ),the string tension density(λ), the expansion (θ) and shear (σ)are given by

$$\rho=\frac{3R}{4T^{3/2}}+M \qquad (37)$$

$$\lambda=0 \qquad (38)$$

$$\theta=\frac{3}{2T^{3/2}}\left(\frac{4T^2}{l}+\frac{4MT^3}{3}+\frac{L^2}{3}+RT^{3/2}\right)^{\frac{1}{2}} \qquad (39)$$

$$\sigma^2=\frac{3}{4}L^2T^{-3} \qquad (40)$$

In the absence of bulk viscosity i.e. M=0 and the presence of magnetic field, the energy density (ρ) the string tension density (λ), the expansion (θ) are given by

$$\rho=\frac{2k}{T^2}+\frac{3R}{4T^{3/2}} \qquad (41)$$

$$\lambda=-\frac{2k}{T^2} \qquad (42)$$

$$\theta=\frac{3}{2T^{3/2}}\left(\frac{4T^2}{l}+\frac{L^2}{3}+RT^{3/2}+4kT\right)^{\frac{1}{2}} \qquad (43)$$

In the absence of magnetic field and bulk viscosity i.e. $k=0, M=0$, the energy density (ρ), the string tension density (λ), the expansion (θ) are given by

$$\rho=\frac{3R}{4T^{3/2}} \qquad (44)$$

$$\lambda=0 \qquad (46)$$

$$\theta = \frac{3}{2T^{3/2}}\left(\frac{4T^2}{l} + \frac{L^2}{3} + RT^{3/2}\right)^{\frac{1}{2}} \tag{47}$$

Conclusion:

For the model (31) the spatial volume $V^3 \to \infty$ as $T \to \infty$. The model (31) starts with a big bang at $T = 0$ and the expansion in the model decreases as time increases.

The energy density $\rho \to \infty$ as $T \to 0$ and ρ is constant as $T \to \infty$.The string tension is negative in the presence of magnetic field and hence geometric p-string model are not possible in the presence of magnetic field. Since $\lim_{T\to\infty} \frac{\sigma}{\theta} = 0$, hence the model isotropizes for large value of T. The model (31) has real physical singularity at $T = 0$.The space time (31) is conformal flat for large value of T.

Next in the presence of bulk viscosity and in the absence of magnetic field the energy density $\rho \to \infty$ as $T \to 0$ and ρ is constant as $T \to \infty$.Also in this case the string tension density $\lambda \to 0$. This gives disagreement with the result shown by Bali and Singh (2005) for Bianchi type V metric in the presence of bulk viscous fluid.

Lastly in the absence of magnetic field and bulk viscosity (i.e. $k = 0$ and $M = 0$),the reality condition $\rho \succ 0$ is satisfied ,when $T \to 0$ then $\rho \to \infty$ and when $T \to \infty$ then $\rho \to 0$.The model starts with a big bang at $T \to 0$ and the expansion in the model decreases as time increases. The space-time is conformally flat for large values T.Since $\lim_{T\to\infty} \frac{\sigma}{\theta} = 0$, hence the model isotropizes for large values of T. Also in this case the string tension density $\lambda \to 0$.

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